Exam Symmetry in Physics

Date	April 20, 2012
Room	X 5113.0201
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the three exercises have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider the dihedral group D_4: $\text{gp}\{c, b\}$ with $c^4 = b^2 = (bc)^2 = e$.

(a) Argue, using geometrical arguments, that D_4 has five conjugacy classes:

$$(e), (c), (c^2), (b), (bc)$$

(b) Determine the dimensions of all inequivalent irreps of D_4.

(c) Construct the character table of D_4.

(d) Identify the proper invariant subgroup(s) of D_4.

(e) Determine the group structure of D_4/H for all invariant subgroups obtained in (d).

(f) Give the explicit matrices of the vector representation D^V for the group elements c, c^2, b, bc. Here D_4 is viewed as a subgroup of the rotations in three dimensions.

(g) Decompose D^V into irreps of D_4 and use this to conclude whether D_4 allows in principle for an invariant three-dimensional vector, such as an electric dipole moment.

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Exercise 2
Consider the orthogonal group in three dimensions: $O(3)$.

(a) Show that the Kronecker delta δ_{ij} is invariant under $O(3)$ transformations.

(b) Determine the subgroup of $O(3)$ transformations that leave the tensor $\sigma_{ij} = \delta_{ij} + a\delta_{i3}\delta_{j3}$ invariant, for nonzero $a \in \mathsf{R}$.

(c) Explain what is an axial vector and a pseudoscalar, and give an example of each.

(d) Determine the subgroup of $O(3)$ transformations that leave invariant the following Hamiltonian:

$$H = \frac{\vec{p}^{\,2}}{2m} + V(|\vec{r}|) + \mu\vec{L}\cdot\vec{S} + \lambda\vec{S}\cdot\vec{B},$$

where μ, λ are numbers and B is a fixed uniform external magnetic field.

Consider the following mapping from $O(3)$ to a subgroup H of $O(3)$:

$$\phi : O \mapsto \det(O)O$$

e) Determine the subgroup H.

f) Show that ϕ is a homomorphism and determine its kernel.

Exercise 3
Consider the special unitary group $SU(3)$.

(a) What is the definition of $SU(3)$?

(b) Give the definition of a Lie algebra.

(c) Count the dimension of the Lie algebra $su(3)$, for example by considering elements of $SU(3)$ close to the identity.

(d) Show whether $SO(3)$ is an invariant subgroup of $SU(3)$ or not.

(e) Show whether $SU(3)$ has an $SU(2)$ subgroup or not.